Recognizing Quadratic Functions and Interpreting Graphs

Teacher	Educator
School	ULAT Partner
Date	October 23, 2026
Duration	47 minutes
Grade Level	Algebra 2, Grades 10–11
Instructional Model	Warm-up, Instructional Ativities, Lesson Synthesis, Cool-Down
SEL Focus	None

Section 1: 10-Component Plan

1. Title

Recognizing Quadratic Functions and Interpreting Graphs

2. Standards

- **HSF-IF.B.4:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
- **HSF-IF.C.7.A:** Graph linear and quadratic functions and show intercepts, maxima, and minima.
- **HSF-IF.B.5:** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

3. SEL Inquiry Aim

How does recognizing the shape and key features of a parabola help us describe real-world motion and make predictions?

4. Objectives

- Identify a quadratic function from its equation, table, or graph and explain why its graph is a parabola.
- Interpret the graph of $y = x^2 - 4$ to state the vertex, zeros, domain, and range.
- Use a contextual parabolic model of a ball's path to connect key features to real-world meaning.
- Analyze and correct an incorrect graph by comparing key features to the correct equation.

5. Vocabulary Bridge

English Term	Definition (English)
quadratic function	A function that can be written in the form $f(x) = ax^2 + bx + c$ where $a \neq 0$; its graph is a parabola.
parabola	The U-shaped curve that is the graph of a quadratic function.
vertex	The highest or lowest point of a parabola; the turning point of the graph.
zeros	The x-values where the function equals zero; the x-intercepts of the graph.
domain	The set of all possible input values (x-values) of a function.
range	The set of all possible output values (y-values) of a function.
axis of symmetry	The vertical line that divides a parabola into two mirror-image halves; it passes through the vertex.

6. Materials

- Projector or document camera to display the rendered graph of $y = x^2 - 4$ and the ball-path illustration
- Printed worksheet with embedded rendered graphs (graph of $y = x^2 - 4$ and the contextual ball-path parabola)
- Graph paper and rulers for student work
- Colored pencils or highlighters for marking vertex and zeros
- Student work space boxes printed on the worksheet
- Exit ticket slip with the supplied graph of $y = x^2 - 4$

7. Lesson Flow

0-3 min

Teacher: Display the warm-up prompt: 'A ball is kicked and its height follows a curve. What shape does the path make, and where does the ball reach its highest point?' Show the contextual illustration of a ball following a parabolic path (a soccer ball launched from ground level, rising to a peak, and landing). Ask students to turn and talk, then record one observation about the shape and one about the highest point. Do not yet name the vocabulary.

Students: Observe the ball-path illustration, discuss with a partner, and write one sentence describing the shape and one sentence about the highest point of the path.

Key Questions:

- What shape does the ball's path make?
- Where on the path is the ball highest, and what does that point represent?

3-13 min

Teacher: Introduce the vocabulary: quadratic function, parabola, vertex, zeros, domain, range, axis of symmetry. Display the rendered coordinate graph of $y = x^2 - 4$ for $-4 \leq x \leq 4$ and $-5 \leq y \leq 13$ with labeled axes, grid, vertex $(0, -4)$, and zeros $(-2, 0)$ and $(2, 0)$. Model reading the graph: trace the parabola, mark the vertex, mark the zeros, and state the domain and range. Emphasize that the vertex is the minimum because the parabola opens upward. Connect back to the ball-path illustration: the vertex is the peak height, the zeros are where the ball touches the ground.

Students: Copy the vocabulary into notes. On the printed graph, highlight the vertex and zeros in different colors. Write the domain and range in the work space and annotate why the range starts at -4.

Key Questions:

- Why is the vertex at $(0, -4)$ and not somewhere else?
- How do the zeros $(-2, 0)$ and $(2, 0)$ connect to the equation $y = x^2 - 4$?
- Why is the range $y \geq -4$ rather than all real numbers?

13-41 min

Teacher: Launch the Explore phase with three differentiated task sets printed on the worksheet, each with its own embedded rendered graph. Task Set A (support): students read the supplied graph of $y = x^2 - 4$ to state vertex, zeros, domain, and range, with sentence frames. Task Set B (core): students analyze a second rendered parabola and identify an incorrect graph among two options by comparing vertex and zeros. Task Set C (extension): students interpret the contextual ball-path parabola, state a reasonable domain and range in context, and explain what the vertex and zeros mean. Circulate, ask probing questions, and pull a small group for Task Set A. Provide worked teacher answers on the key for all three sets.

Students: Work through the assigned task set in the printed work space. Mark vertex and zeros on each embedded graph, write domain and range, and justify the choice of the correct graph. Students in Task Set C write a short contextual explanation of the vertex and zeros.

Key Questions:

- Which key features let you tell a correct graph from an incorrect one?
- What is the domain and range of this parabola, and how do you know from the graph?
- In the ball-path model, what does the vertex represent and what do the zeros represent?

41-43 min

Teacher: Facilitate the Lesson Synthesis: display the rendered graph of $y = x^2 - 4$ again and have students share how they found the vertex, zeros, domain, and range. Record student reasoning on the board and connect the incorrect-graph analysis to the key features. Summarize that a quadratic graph is a parabola whose vertex, zeros, domain, and range can be read directly from the graph and verified from the equation.

Students: Share reasoning with the class, compare answers, and correct any misconceptions in their notes. Add one sentence to notes explaining how to check a graph against its equation.

Key Questions:

- How can you verify a graph is correct using the equation?
- What is the range of $y = x^2 - 4$ and why?

43-47 min

Teacher: Distribute the exit ticket with the supplied rendered graph of $y = x^2 - 4$. Instruct students to identify the vertex and zeros from the graph and justify the range in one or two sentences. Collect tickets and use the answer key to check success criteria.

Students: Independently use the supplied graph to write the vertex, the zeros, and a justification of the range in the work space on the exit ticket.

Key Questions:

- What are the vertex and zeros of the supplied graph?
- Why is the range of $y = x^2 - 4$ equal to $y \geq -4$?

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8. Differentiation Overview

- **ELL:** Provide a labeled visual glossary with the rendered graph of $y = x^2 - 4$ and arrows pointing to vertex, zeros, and axis of symmetry; pair each term with a sentence frame such as 'The vertex is ___ because ___.'
- **IEP:** Provide the rendered graphs with pre-marked grid lines and a partially completed work space; chunk Task Set A into three short steps with checkpoints and allow oral responses recorded by the teacher.
- **Advanced:** Assign Task Set C with the contextual ball-path parabola; ask students to justify a reasonable domain and range in context and to explain how changing the constant term would shift the vertex and zeros.
- **Struggling:** Use Task Set A with the supplied rendered graph, color-code the vertex and zeros, and use a guided checklist: find the lowest point, find where the graph crosses the x-axis, then read the y-values for the range.

9. Assessment Plan

Formative: During Explore, the teacher circulates and checks each student's marked vertex and zeros on the embedded graphs and their written domain and range; probing questions target the justification of the range and the incorrect-graph analysis.

Summative: The exit ticket uses the supplied rendered graph of $y = x^2 - 4$ to assess identifying the vertex and zeros and justifying the range, scored against the success criteria.

10. Cool-down

Use the supplied graph of $y = x^2 - 4$ to identify the vertex and the zeros, then justify in one or two sentences why the range is $y \geq -4$.

Success Criteria:

- States the vertex as $(0, -4)$.
- States the zeros as $(-2, 0)$ and $(2, 0)$.
- Justifies the range as $y \geq -4$ because the lowest point of the parabola is the vertex at $y = -4$ and the parabola opens upward.

Section 2: Global Context

At a community track meet in a diverse neighborhood, a student launches a shot put. The height of the shot put above the ground can be modeled by a quadratic function. Suppose the height in meters after t seconds is given by $h(t) = -5t^2 + 20t + 1.5$. At $t = 0$, the height is 1.5 meters. The shot put reaches its maximum height at the vertex. To find the vertex, we use $t = -\frac{b}{2a} = -\frac{20}{2(-5)} = 2$ seconds. The maximum height is $h(2) = -5(4) + 20(2) + 1.5 = -20 + 40 + 1.5 = 21.5$ meters. The shot put hits the ground when $h(t) = 0$. Solving $-5t^2 + 20t + 1.5 = 0$ gives approximately $t \approx 4.07$ seconds. This parabolic path is similar to the graph of $y = x^2 - 4$, which opens upward, while the shot put's path opens downward because $a < 0$. In this lesson, we will explore how to recognize quadratic functions from their equations and graphs, interpret key features such as vertex, zeros, domain, and range, and identify incorrect graphs. We will also analyze a ball's parabolic trajectory and use the graph of $y = x^2 - 4$ to answer questions about its vertex, zeros, and range.

Hook: How can a single equation predict the exact peak height and landing time of a thrown object?

Section 3: Differentiated Packets

Packet A

Word Bank: vertex, parabola, zeros, domain, range, axis of symmetry, quadratic function, minimum value

Question 1. Look at the graph of $y = x^2 - 4$ shown below. Identify the vertex and the zeros (where the graph crosses the x-axis). [GRAPH: Coordinate plane showing parabola $y = x^2 - 4$ for $-4 \leq x \leq 4$ and $-5 \leq y \leq 13$. Grid lines present. Vertex labeled at $(0, -4)$. Zeros marked at $(-2, 0)$ and $(2, 0)$. Axes labeled with x-axis from -4 to 4 and y-axis from -5 to 13.]

- The vertex is the lowest point on this parabola, located at the ordered pair _____ , _____.
- The zeros are where the graph crosses the x-axis, which happens at _____ and _____.
- The vertex tells us that the minimum y-value is _____ when $x =$ _____.
- This parabola opens upward because the coefficient of x^2 is _____ (positive or negative).

Answer: The vertex is at $(0, -4)$. This is the lowest point on the parabola. The zeros are at $x = -2$ and $x = 2$. These are the points where the graph crosses the x-axis, meaning $y = 0$ at those locations. We can verify: when $x = -2, y = (-2)^2 - 4 = 4 - 4 = 0$. When $x = 2, y = (2)^2 - 4 = 4 - 4 = 0$. The parabola opens upward because the coefficient of x^2 is positive (it is $+1$).

High-support students often confuse the vertex with one of the zeros. Reinforce that the vertex is ONE point with both an x-coordinate and y-coordinate. Zeros are x-intercepts only (y = 0 always). Common error: reading $(0, -4)$ as two separate zeros instead of one turning point.

Question 2. For the function $y = x^2 - 4$, identify the domain and range. Use the graph to justify your answer. [GRAPH: Same coordinate plane as Problem 1, showing parabola $y = x^2 - 4$ with vertex at $(0, -4)$ and zeros at $(-2, 0)$ and $(2, 0)$.]

- The domain is all possible _____ -values. For this parabola, x can be any real number, so domain = _____ .
- The range is all possible _____ -values. The parabola's lowest point is at $y = -4$, so $y \geq$ _____ .
- Looking at the graph, the parabola opens _____ and never goes below the vertex.
- In interval notation, the range is _____ ∞) or in inequality form, $y \geq$ _____ .

Answer: Domain: All real numbers, or $\mathbb{R}$, or $(-\infty, \infty)$. The graph extends left and right infinitely. Range: $y \geq -4$, or in interval notation $[-4, \infty)$. The parabola opens upward and has a minimum at the vertex, which is at $y = -4$. No y-values below -4 appear on the graph, but the parabola reaches all y-values from -4 and above.

Students often write the range incorrectly by not recognizing the minimum point. Common error: writing range as $(-4, \infty)$ with a parenthesis instead of a bracket at -4. Since $y = -4$ IS achieved at the vertex, we use a bracket: $[-4, \infty)$.

Question 3. A basketball player shoots a ball that follows a parabolic path. The height h (in meters) of the ball at time t (in seconds) is given by $h(t) = -5t^2 + 10t + 2$. Which of the following is the best description of the graph of this function? (A) Opens upward with a maximum height of 7 meters (B) Opens downward with a maximum height of 7 meters (C) Opens upward with a minimum height of 2 meters (D) Opens downward with a minimum height of 2 meters [VISUAL: Illustration of a basketball player shooting a ball in a parabolic arc, with the ball at various points along the curved path.]

- The coefficient of t^2 is _____ , which is negative, so the parabola opens _____ .
- When a parabola opens downward, it has a _____ (maximum or minimum) point at the vertex.
- For a real-world ball toss, the graph starts at ground level or above, rises to a peak, then _____ .
- The correct answer is (_____) because the parabola opens _____ and the highest point is called a maximum.

Answer: The correct answer is (B) Opens downward with a maximum height of 7 meters. The coefficient of t^2 is -5, which is negative. When the leading coefficient is negative, the parabola opens downward and has a maximum value at its vertex. We can find the vertex using $t = -\frac{b}{2a}$. Here, $a = -5$ and $b = 10$, so $t = -\frac{10}{2(-5)} = -\frac{10}{-10} = 1$. At $t = 1, h(1) = -5(1)^2 + 10(1) + 2 = -5 + 10 + 2 = 7$. So the maximum height is 7 meters at time $t = 1$ second. This makes physical sense: the ball is launched, reaches a peak of 7 meters, then falls back down.

Common error: Students choose (A) because they see 'maximum' and 'height' without checking the sign of the leading coefficient. Reinforce: NEGATIVE leading coefficient = opens DOWN = has a maximum. POSITIVE leading coefficient = opens UP = has a minimum. The real-world context (a ball path) should also guide intuition.

Question 4. Look at the four graphs below. Three graphs represent quadratic functions. One graph is NOT a quadratic function. Identify which graph is not quadratic and explain why. [GRAPH SET: Four coordinate planes labeled A, B, C, D: A: Parabola opening upward with vertex at (1, 0) B: Straight line with slope 2 passing through origin C: Parabola opening downward with vertex at (-1, 3) D: Parabola opening upward with vertex at (0, -2)] All axes labeled -5 to 5.

- Graphs A, C, and D show _____ shapes (curved/straight). Graph B shows a _____ shape (curved/straight).
- A quadratic function has a graph that is always a _____ , which is U-shaped or upside-down U-shaped.
- Graph B is not a quadratic function because it is a _____ line, not a parabola.
- The answer is _____ because it does not have the curved parabolic shape required for a quadratic function.

Answer: Graph B is NOT a quadratic function. Graphs A, C, and D all show parabolas (curved, U-shaped or inverted U-shaped), which are characteristic of quadratic functions. Graph B shows a straight line, which is a linear function (degree 1), not a quadratic function (degree 2). Quadratic functions always produce parabolic graphs; linear functions produce straight lines. Since Graph B is a line, it cannot represent a quadratic function.

Common error: Students may confuse 'parabola' with other curve shapes or not recognize that straight lines are never quadratic. Reinforce the visual: quadratic = always curved like a U or ∩. Line = always straight = NOT quadratic. Use the word bank term 'parabola' consistently.

Packet B

Question 1. The graph of $y = x^2 - 4$ is shown below. Identify the vertex, the zeros (x-intercepts), and explain why the y-intercept is located at $(0, -4)$. [GRAPH: A parabola opening upward. The vertex is at $(0, -4)$. The graph extends from $x = -4$ to $x = 4$ on the x-axis and from $y = -5$ to $y = 13$ on the y-axis. Grid lines are visible. The parabola intersects the x-axis at $(-2, 0)$ and $(2, 0)$. The parabola intersects the y-axis at $(0, -4)$. The vertex is marked with a dot and labeled. The zeros are marked with dots and labeled.]

Answer: Vertex: $(0, -4)$ (the lowest point on the parabola, also called the minimum). Zeros (x-intercepts): $x = -2$ and $x = 2$ (where the parabola crosses the x-axis). The y-intercept occurs when $x = 0$. Substituting into the equation: $y = (0)^2 - 4 = -4$. Therefore, the y-intercept is $(0, -4)$, which is also the vertex in this case because the parabola is symmetric about the line $x = 0$.

Common error: Students may confuse the vertex with the y-intercept in all cases. Clarify that the y-intercept is always found by setting $x = 0$, while the vertex is the turning point and is found at the axis of symmetry. In this example they coincide, but this is not always true.

Question 2. Use the graph of $y = x^2 - 4$ to determine the domain and range of this function. Justify your answers by explaining what domain and range represent.

Answer: Domain: All real numbers, or $(-\infty, \infty)$. Justification: The parabola extends infinitely to the left and right. For any real number x that you input, the function produces a valid output. Range: $[-4, \infty)$ or $y \geq -4$. Justification: The parabola opens upward with a minimum point (vertex) at $(0, -4)$. The y-values start at -4 and increase without bound as x moves away from the vertex in either direction. The parabola never goes below $y = -4$.

Common error: Students may state the range as $(-\infty, \infty)$ if they fail to recognize that the parabola has a minimum value. Emphasize that the minimum y-value on the graph is -4 (the vertex), so the range cannot include values below this.

Question 3. A ball is kicked from ground level and follows a parabolic path modeled by the equation $h(t) = -16t^2 + 32t$, where h is height in feet and t is time in seconds. Find the zeros of this function and interpret what each zero represents in the context of the ball's motion. [ILLUSTRATION: A sketch of a ball's parabolic trajectory. The ball starts at ground level (origin), rises to a maximum height, and returns to ground level. The path is labeled with the vertex marking the maximum height and the zeros marking where the ball leaves the ground and where it lands.]

Answer: To find the zeros, set $h(t) = 0$: $-16t^2 + 32t = 0$. Factor out $-16t$: $-16t(t - 2) = 0$. This gives $t = 0$ or $t = 2$. The zeros are $t = 0$ seconds and $t = 2$ seconds. Interpretation: At $t = 0$, the ball is at ground level (initial position). At $t = 2$, the ball returns to ground level after reaching a maximum height. The ball is in the air for 2 seconds.

Common error: Students may forget to set the function equal to zero or may fail to factor correctly. Remind them that zeros are the values where the output equals zero (in this case, when height equals zero, or ground level). Also note that only the positive time values make sense in this context; negative time is not physically meaningful.

Question 4. Three graphs are shown below. Graph A shows $y = x^2 - 4$, Graph B shows $y = x^2 + 1$, and Graph C is incorrectly labeled as $y = x^2 - 4$ but has a vertex at $(0, 4)$. Identify which graph is incorrect and explain why the vertex location contradicts the equation. [GRAPH A: A parabola opening upward with vertex at $(0, -4)$, zeros at $(-2, 0)$ and $(2, 0)$. Grid and axes are labeled.] [GRAPH B: A parabola opening upward with vertex at $(0, 1)$, no x-intercepts visible. Grid and axes are labeled.] [GRAPH C: A parabola opening upward with vertex at $(0, 4)$, labeled as $y = x^2 - 4$. Grid and axes are labeled.]

Answer: Graph C is incorrect. If the equation is $y = x^2 - 4$, then the vertex should be at $(0, -4)$, not $(0, 4)$. To verify: when $x = 0$, $y = (0)^2 - 4 = -4$. Graph C shows a vertex at $(0, 4)$, which would correspond to an equation like $y = x^2 + 4$, not $y = x^2 - 4$. The constant term in the equation determines the vertical shift of the parabola. A negative constant (-4) shifts the parabola down 4 units, while a positive constant ($+4$) shifts it up 4 units.

Common error: Students may confuse the sign of the constant term and its effect on the vertex. Reinforce that the y-coordinate of the vertex for $y = a(x - h)^2 + k$ is k. In standard form $y = x^2 - 4$, we can rewrite as $y = (x - 0)^2 - 4$, so the vertex is at $(0, -4)$, not $(0, 4)$.

Packet C

Question 1. A basketball player shoots a ball that follows a parabolic path modeled by the function $h(t) = -16t^2 + 32t + 6$, where h is the height in feet and t is time in seconds. (a) Analyze the vertex of this quadratic and explain what it represents in the context of the basketball shot. (b) Evaluate whether the ball will reach a height of 25 feet. Justify your reasoning using the quadratic formula or by completing the square. (c) Synthesize the features of the parabola (vertex, zeros, axis of symmetry, and direction of opening) to propose a real-world constraint that would make this model invalid for all positive values of t.

Answer: (a) The vertex form can be found by completing the square or using $t = -\frac{b}{2a} = -\frac{32}{2(-16)} = 1$. At $t = 1$, $h(1) = -16(1)^2 + 32(1) + 6 = -16 + 32 + 6 = 22$ feet. The vertex is $(1, 22)$, which represents the maximum height of 22 feet reached at 1 second after the shot. (b) Set $-16t^2 + 32t + 6 = 25$. Rearranging: $-16t^2 + 32t - 19 = 0$, or $16t^2 - 32t + 19 = 0$. Using the discriminant: $\Delta = (-32)^2 - 4(16)(19) = 1024 - 1216 = -192 < 0$. Since the discriminant is negative, there are no real solutions. The ball will not reach 25 feet because the maximum height is 22 feet, which is less than 25 feet. (c) The zeros of the function represent times when the ball is at ground level ($h = 0$). Solving $-16t^2 + 32t + 6 = 0$: $t = \frac{-32 \pm \sqrt{1024 - (-384)}}{-32} = \frac{-32 \pm \sqrt{1408}}{-32} \approx \frac{-32 \pm 37.53}{-32}$. This gives $t \approx 2.11$ seconds (taking the positive root). A real-world constraint is that the parabola becomes invalid after the ball hits the ground or another surface; the model assumes the ball travels through empty space, but in reality, it cannot descend below ground level ($h = 0$). Therefore, the domain must be restricted to $0 \leq t \leq 2.11$ seconds.

An exemplary response recognizes that the vertex represents the peak of the trajectory and correctly interprets the discriminant to determine feasibility. The student should justify why the ball cannot exceed 22 feet by comparing the given height to the vertex value. Part (c) demonstrates synthesis by connecting the mathematical features (zeros, parabolic shape) to a physical constraint, showing that models have limited domains in real-world contexts.

Question 2. Consider the graph of $y = x^2 - 4$ shown below. Your classmate claims that the range of this function is $y \geq -4$ because 'the vertex is at the minimum, so all outputs are greater than or equal to the y-coordinate of the vertex.' (a) Evaluate your classmate's claim and justify whether it is correct or incorrect using properties of parabolas. (b) Address the counterargument that 'the range could be all real numbers because the parabola goes up forever.' Explain why this counterargument is valid or invalid. (c) Write a brief explanation (3-4 sentences) that you could give to your classmate to help them understand the correct range and why the vertex location determines it.

Answer: (a) The classmate's claim is correct. The function $y = x^2 - 4$ is a parabola that opens upward (since the coefficient of x^2 is positive). The vertex is at $(0, -4)$, which is the minimum point on the graph. Since the parabola opens upward, all points on the curve have y-values greater than or equal to the minimum y-value at the vertex. Therefore, the range is $y \geq -4$ or $[-4, \infty)$. (b) The counterargument is partially valid but incomplete. It is true that the parabola extends upward infinitely (to the right and left of the vertex), so outputs do increase without bound. However, this does not mean the range is all real numbers. The critical feature is that the parabola has a minimum value: no output is less than -4. The counterargument correctly identifies that y can be arbitrarily large, but it overlooks the lower bound created by the vertex. A more complete statement would be: 'The parabola goes up forever, so there is no upper limit to the range, but the vertex creates a lower limit.' (c) 'The range of a parabola is determined by the vertex, which is the lowest or highest point on the curve. In this function, the vertex at $(0, -4)$ is the minimum point because the parabola opens upward. This means no output value can be less than -4, but the parabola continues upward forever, so larger values are always possible. Therefore, the range includes -4 and all values above it: $y \geq -4$.'

An exemplary response demonstrates understanding of how the direction of opening and vertex location jointly determine the range. Students should recognize that the counterargument contains a kernel of truth (unbounded growth) but misses the essential constraint (the minimum). The written explanation in part (c) should be clear enough for a peer to understand, avoiding jargon and using the graph as a reference.

Question 3. A school fundraiser sells T-shirts. The profit function is $P(x) = -2(x - 30)^2 + 1800$, where x is the number of shirts sold and $P(x)$ is the profit in dollars. (a) Analyze the vertex and zeros of this quadratic to determine the break-even points and maximum profit. (b) Evaluate the validity of the statement: 'If we sell 60 shirts, we will make more profit than if we sell 50 shirts, because 60 is closer to the vertex.' Use calculations to justify your answer. (c) Propose a domain restriction for this function that reflects real-world constraints. Justify your restriction by considering both the mathematical features of the parabola and practical business considerations (e.g., inventory, market size).

Answer: (a) The function is in vertex form $P(x) = -2(x - 30)^2 + 1800$. The vertex is $(30, 1800)$, meaning the maximum profit of $1800 dollars is achieved when 30 shirts are sold. To find break-even points, set$P(x) = 0$: -2(x - 30)^2 + 1800 = 0 \Rightarrow (x - 30)^2 = 900 \Rightarrow x - 30 = \pm 30 \Rightarrow x = 0 or x = 60. The break-even points are at$x = 0$($no shirts sold, no profit or loss$) and$x = 60$($selling 60 shirts results in zero profit$). Between these points, profit is positive. (b) The statement is incorrect. While 60 is farther from the vert$P(50) = -2(50 - 30)^2 + 1800 = -2(400) + 1800 = -800 + 1800 = 1000 dollars.$P(60) = -2(60 - 30)^2 + 1800 = -2(900) + 1800 = -1800 + 1800 = 0 dollars. Since$P(50) = 1000 > P(60) = 0$, selling 50 shirts yields more profit than selling 60 shirts. The statement confuses 'proximity to the vertex' with 'higher profit value'; being clos$0 \le x \le 60$($and possibly tighter, e.g.,$0 \le x \le 50$). Mathematically, this range includes all break-even and profitable sales levels; profit is zero or positive only when$0 \le x \le 60$. Practically, a school would only want to sell shirts while generating profit or at least breaking even. Additionally, the school likely has inventor$0 \le x \le 50$ ensures profit stays above 1000 dollars while accounting for potential supply chain challenges.

An exemplary response demonstrates facility with vertex form, accurate computation of break-even points, and recognition that proximity to the vertex does not automatically ensure higher profits (the shape of the parabola matters). Part (b) requires explicit calculation to refute the student's faulty reasoning. Part (c) shows synthesis by combining the mathematical constraints (zeros, profit positivity) with realistic business reasoning, demonstrating that domain restrictions in modeling problems reflect both mathematics and context.

Question 4. Below is the graph of $y = x^2 - 4$ for $-4 \leq x \leq 4$. Your task is to identify and explain why the following statement is false: 'The domain of this function is $-4 \leq x \leq 4$ because those are the x-values shown on the graph.' Use the graph and your understanding of domain and range to justify your answer. Then, propose what the actual domain and range of $y = x^2 - 4$ are, assuming no additional restrictions are given.

Answer: The statement is false because the domain is not restricted to the x-values displayed on the graph. The domain of a function is the set of all possible input values for which the function is defined. The quadratic function $y = x^2 - 4$ is a polynomial and is defined for all real numbers unless explicitly restricted. The graph shown only displays a window from -4 to 4 on the x-axis for visualization purposes, but this does not mean the function only exists in that interval. For example, $y = (5)^2 - 4 = 21$ is a valid output; the point $(5, 21)$ is on the function even though it is not shown on the displayed graph. The actual domain of $y = x^2 - 4$ is all real numbers, written as $(-\infty, \infty)$ or $\mathbb{R}$, since any real number can be substituted for x and a valid y-value will result. The range, determined by the vertex $(0, -4)$ and the upward-opening parabola, is $[-4, \infty)$ or $y \geq -4$. The graph window (viewing window) is distinct from the domain and range; the viewing window is a choice made for clarity and convenience, while domain and range are mathematical properties of the function itself.

An exemplary response clearly distinguishes between the viewing window (the portion of the graph displayed) and the domain of the function (all allowable inputs). Students should recognize that a graph is a visual tool with practical limitations, but the mathematical function extends beyond what is shown. The response should use concrete examples (e.g., testing a point outside the window) to illustrate why the domain is larger than the displayed interval.

Section 4: Teacher Toolkit

3-Point Mastery Rubric

3 — Complete Mastery: Student correctly identifies the vertex $(0, -4)$ and both zeros $(-2, 0)$ and $(2, 0)$ from the graph and justifies the range as $y \geq -4$ because the parabola opens upward with a minimum at the vertex. Student accurately distinguishes a correct quadratic graph from an incorrect one and explains the reasoning using key features such as vertex, zeros, and direction of opening.

2 — Partial Understanding: Student identifies the vertex and zeros correctly but gives an incomplete or imprecise justification for the range, such as stating the range without connecting it to the vertex as a minimum. Student may identify an incorrect graph but offers only a partial explanation that references one feature rather than comparing multiple key features.

1 — Needs Significant Support: Student struggles to locate the vertex or zeros accurately and cannot justify the range using the graph's minimum point. Student is unable to distinguish a correct quadratic graph from an incorrect one or provides no mathematical reasoning for the choice.

Common Misconceptions

Range equals domain

Students think: Students think the range of a quadratic function is the same as its domain, so they write the range as all real numbers or copy the domain interval directly.

Why: They conflate the horizontal input values with the vertical output values and do not connect the graph's lowest point to the set of possible y-values.

Fix: Have students trace the graph vertically with a finger and mark the lowest y-value reached, then state that all y-values at or above that minimum are outputs, writing the range as $y \geq -4$.

Zeros confused with vertex

Students think: Students point to the vertex $(0, -4)$ as a zero because it is the lowest point on the graph and looks important.

Why: They do not yet distinguish between where the graph crosses the x-axis (zeros) and where it turns (vertex), often because they read coordinates without connecting them to axis crossings.

Fix: Use color-coded annotations: circle the x-axis crossings in one color and label the turning point in another, then ask students to state what makes a point a zero (its y-coordinate is 0).

Ignoring direction of opening

Students think: Students assume every parabola has a minimum and a range of $y \geq k$, even when the graph opens downward.

Why: They overgeneralize from the first example and do not check the sign of the leading coefficient or the visual direction of the parabola.

Fix: Present side-by-side graphs of $y = x^2 - 4$ and $y = -x^2 + 4$ and have students compare the vertex type (minimum versus maximum) and write each range, reinforcing that the sign of a determines whether the range is $y \geq k$ or $y \leq k$.

Implementation Notes

- Setup: Before class, print the student packet double-sided so the rendered graph of $y = x^2 - 4$ and the ball-path illustration appear on the front page, and display the same graph on the projector for the warm-up and cool-down. Have colored pencils or highlighters available at each table so students can mark the vertex and zeros directly on the printed graph. Post the 47-minute timer visibly and label the four lesson phases on the board.
- Pacing: Launch with a 3-minute warm-up asking students to sketch any parabola and name one feature, then move into the 28-minute Explore where students rotate through the three differentiated task sets in tiered order. Reserve exactly 12 minutes for Synthesis to compare the correct graph with the incorrect graph and formalize domain, range, vertex, and zeros, and protect the final 4 minutes for the cool-down using the supplied graph. Announce a 2-minute warning before each transition so groups can finish writing in the work space.
- Grouping: Assign students to mixed-ability triads for the Explore phase, with each triad starting on Task Set A and progressing to B and C as readiness allows. Give each triad a role card (Graph Reader, Recorder, Checker) and rotate roles at the midpoint so all students practice interpreting the graph. Circulate with the teacher answer key to ask probing questions about why the range starts at -4 rather than simply confirming answers.
- Tiered packet flow: Task Set A focuses on reading the vertex and zeros directly from the labeled graph and completing sentence frames for domain and range. Task Set B asks students to justify the range and compare two graphs to identify which one is incorrect, citing at least two features. Task Set C extends to writing the range in interval notation and explaining how changing the constant term shifts the vertex and zeros, with the teacher answer key showing full worked solutions for all three sets.

Extension Activities

- Early finisher task: Give students a blank grid and ask them to write the equation of a quadratic whose vertex is $(0, 3)$ and whose zeros are approximately $(-1.7, 0)$ and $(1.7, 0)$, then verify by graphing and stating the domain and range. Students can check their work against a partner and explain how the constant term controls the vertical shift.
- Cross-curricular connection: Connect the parabolic path of a thrown ball to a physics discussion of projectile motion, where the height-versus-time graph is quadratic and the vertex represents maximum height. Students write a short paragraph explaining what the vertex and zeros mean physically in the ball illustration, linking mathematics to kinematics.
- Real-world application: Have students model the height of a water fountain arc or a basketball shot using a quadratic function, then identify the vertex as the maximum height and the zeros as the points where the object reaches ground level. Students present their model to the group and justify the domain and range in context, including why negative time or negative height may be excluded.