Teacher source review

- Original instructional examples may contain model-generated claims; review before teaching.

Solve Quadratic Equations by Factoring and the Zero-Product Property

Teacher	Educator
School	ULAT Partner
Date	October 26, 2026
Duration	47 minutes
Grade Level	Algebra II, Grade 11
Instructional Model	Warm-up, Instructional Ativities, Lesson Synthesis, Cool-Down
SEL Focus	None

Section 1: 10-Component Plan

1. Title

Solve Quadratic Equations by Factoring and the Zero-Product Property

2. Standards

- **HSA-REI.B.4b:** Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation.
- **HSA-SSE.B.3a:** Factor a quadratic expression to reveal the zeros of the function it defines.
- **HSA-APR.A.1:** Understand that polynomials form a system closed under addition, subtraction, and multiplication; multiply polynomials fluently.

3. SEL Inquiry Aim

How does checking each factor against the zero-product property help us trust our own reasoning when we solve an equation?

4. Objectives

- Factor quadratic expressions of the form $x^2 + bx + c$ and $ax^2 + bx + c$ with $a > 1$, including expressions that require a greatest common factor first.
- Apply the zero-product property to justify each solution of a factored quadratic equation.

- Analyze an error in which dividing both sides of an equation by x loses a solution, and explain how factoring preserves both solutions.

5. Vocabulary Bridge

English Term	Definition (English)
quadratic equation	An equation that can be written in the form $ax^2 + bx + c = 0$ with $a \neq 0$.
factor	To rewrite an expression as a product of expressions, such as $x^2 - 5x + 6 = (x - 2)(x - 3)$.
zero-product property	If $ab = 0$, then $a = 0$ or $b = 0$; used to set each factor of a factored equation equal to zero.
greatest common factor	The largest expression that divides each term of a polynomial, such as $2x$ in $2x^2 + 6x$.
monic quadratic	A quadratic expression whose leading coefficient is 1, such as $x^2 + 7x + 12$.
leading coefficient	The coefficient of the term with the highest power of the variable, such as 3 in $3x^2 - 10x + 8$.
solution	A value of the variable that makes the equation true.

6. Materials

- Packet A, Packet B, and Packet C practice handouts (one per student, assigned by teacher)
- Writing workspace sheet with sentence stems
- Optional early-finisher challenge card
- Cool-down exit ticket slip
- Whiteboard or chart paper for synthesis
- Timer for segment pacing

7. Lesson Flow

0-3 min

Teacher: Display the warm-up prompt: 'Factor $x^2 + 7x + 12$ and explain how the two numbers you chose connect to 7 and 12.' Give students 2 minutes to write in their workspace, then take 1 minute of quiet think time before collecting responses. Do not distribute the exit ticket.

Students: Students factor the monic quadratic independently in their writing workspace and record one sentence explaining the connection between the factors and the coefficients.

Key Questions:

- Which two numbers multiply to 12 and add to 7?
- How do those two numbers appear in the factored form?

3-13 min

Teacher: Launch the Explore phase. Distribute Packet A, Packet B, or Packet C as assigned. Each packet contains the same three core tasks: a monic quadratic, a quadratic with leading coefficient greater than 1, and a problem requiring a greatest common factor first. Circulate and ask students to justify each factor pair. Provide sentence stems on the board: 'I know the factors are ___ because ___.' and 'The zero-product property tells me ___.' Step-by-step teacher answers are on the answer key for each packet.

Students: Students work in pairs on their assigned packet, factoring each quadratic and writing a justification for each factor pair using the sentence stems.

Key Questions:

- For $3x^2 - 10x + 8$, which factor pair of $a \cdot c$ also combines to give the middle coefficient?
- What is the first step when every term shares a common factor?

13-23 min

Teacher: Continue Explore with the error-analysis task. Display: 'A student solves $x^2 = 5x$ by dividing both sides by x and gets $x = 5$. Is anything missing?' Give pairs 4 minutes to discuss, then 3 minutes to write a corrected solution. Prompt students to name the lost solution and explain why dividing by x removes it. Do not collect or preview the exit ticket.

Students: Students analyze the flawed solution, identify the lost solution $x = 0$, and rewrite the solution by factoring $x^2 - 5x = 0$ as $x(x - 5) = 0$ and applying the zero-product property.

Key Questions:

- What value of x makes the division by x invalid?
- How does factoring avoid losing that solution?

23-33 min

Teacher: Students continue in their packets, completing the remaining practice items that include a monic quadratic, a quadratic with leading coefficient greater than 1, and a greatest-common-factor-first problem. Teacher confers with pairs who need support and asks students to check each solution by substitution. Offer the optional early-finisher challenge card only to students who finish early; it is separate from the exit ticket.

Students: Students complete the remaining packet items, show factorization and zero-product-property justification for each, and check solutions by substitution. Early finishers attempt the optional challenge.

Key Questions:

- How can substitution confirm that both solutions are correct?
- Where does the greatest common factor appear in your factored form?

33-43 min

Teacher: Facilitate the lesson synthesis. Invite two or three pairs to share a factored form and its justification on chart paper. Highlight the three structures: monic quadratics, quadratics with leading coefficient greater than 1, and problems requiring a greatest common factor first. Revisit the error-analysis task and ask the class to state the zero-product property in their own words. Do not distribute the exit ticket yet.

Students: Students share their work, compare strategies across packets, and add a corrected statement of the zero-product property to their notes.

Key Questions:

- How does the zero-product property justify setting each factor equal to zero?
- Why is dividing by a variable risky when solving a quadratic equation?

43-47 min

Teacher: Distribute the exit ticket and read the prompt aloud: 'Solve $x^2 - 5x + 6 = 0$ by factoring. Show the factorization and use the zero-product property to justify both solutions.' Give students 4 minutes to write their response independently. Collect all exit tickets at the end of the segment.

Students: Students write their factorization, set each factor equal to zero, and justify both solutions using the zero-product property.

Key Questions:

- Which two numbers multiply to 6 and add to -5?
- How does the zero-product property give you both solutions?

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8. Differentiation Overview

- **ELL:** Provide sentence stems and a labeled worked example on each packet; pair mathematical symbols with brief plain-language definitions in the vocabulary bridge.

- **IEP:** Offer Packet A with partially completed factor-pair tables and a checklist that prompts students to factor, set each factor to zero, and check by substitution.
- **Advanced:** Offer Packet C with less scaffolding and the optional early-finisher challenge, which asks students to create and solve a quadratic that requires a greatest common factor first and to explain why dividing by the variable would lose a solution.
- **Struggling:** Use Packet A with step-by-step prompts, a multiplication table for factor pairs, and teacher check-ins after each task; emphasize checking solutions by substitution.

9. Assessment Plan

Formative: Teacher circulates during Explore and synthesis, listening for correct factor pairs and zero-product-property justifications; pairs record answers in the writing workspace and share during synthesis.

Summative: The exit ticket is the only summative check for this section and is scored solely on the requested prompt.

10. Cool-down

Solve $x^2 - 5x + 6 = 0$ by factoring. Show the factorization and use the zero-product property to justify both solutions.

Success Criteria:

- Shows the factorization $x^2 - 5x + 6 = (x - 2)(x - 3)$ or an equivalent correct factorization.
- Sets each factor equal to zero, $(x - 2) = 0$ and $(x - 3) = 0$, citing the zero-product property.
- States both solutions, $x = 2$ and $x = 3$, and justifies each one using the zero-product property.

Section 2: Global Context

This section centers on a student-centered exploration of solving quadratic equations by factoring and applying the zero-product property. Students begin with a brief warm-up that asks them to factor $x^2 + 7x + 12$ and explain how the chosen numbers connect to the coefficients 7 and 12, activating prior knowledge about factor pairs. During the Explore phase, students work in pairs on one of three neutral packets, Packet A, Packet B, or Packet C, each preserving the same learning goal while varying the level of scaffolding. Every packet contains a monic quadratic, a quadratic with leading coefficient greater than 1, and a problem requiring a greatest-common-factor first step, and students justify each factor pair using sentence stems such as 'I know the factors are ___ because ___.' and 'The zero-product property tells me ___.' A key error-analysis task asks students to examine a flawed solution of $x^2 = 5x$ in which a student divides both sides by x and obtains only $x = 5$; pairs identify the lost solution $x = 0$ and rewrite the work by factoring $x^2 - 5x = 0$ as $x(x - 5) = 0$. The teacher circulates, confers with pairs, and prompts students to check each solution by substitution, while an optional early-finisher challenge is offered separately to students who finish early. In the Synthesis, pairs share factored forms and justifications, the class compares the three quadratic structures, and students restate the zero-product property in their own words. The Cool-down, which is the only summative check for this section, asks students to solve $x^2 - 5x + 6 = 0$ by factoring, show the factorization, and use the zero-product property to justify both solutions. The lesson will solve the

Cool-down equation $x^2 - 5x + 6 = 0$ by factoring and justifying both solutions with the zero-product property.

Hook: If dividing both sides of $x^2 = 5x$ by x gives only $x = 5$, what happened to the other solution?

Section 3: Differentiated Packets

Packet A

Word Bank: factor, zero-product property, quadratic equation, greatest common factor, monic quadratic, leading coefficient, solution

Question 1. Factor $x^2 + 7x + 12$ and explain how the two numbers you chose connect to 7 and 12.

- I need two numbers that multiply to _____ and add to _____.
- The two numbers I chose are _____ and _____ because _____ times _____ equals 12 and _____ plus _____ equals 7.
- The factored form is $(x + ____)(x + ____)$.
- I can check by expanding: $(x + ____)(x + ____) = x^2 + ____x + ____$.

Answer: The two numbers are 3 and 4. They multiply to 12 and add to 7. The factorization is $x^2 + 7x + 12 = (x + 3)(x + 4)$. Check: $(x + 3)(x + 4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12$.

Common error: Students may find two numbers that multiply to 12 but do not add to 7 (e.g., 2 and 6). Emphasize that BOTH conditions must be satisfied. Monic quadratics have a leading coefficient of 1, so the factor pairs of the constant term are the key.

Question 2. Factor $3x^2 - 10x + 8$ by finding two numbers that multiply to $a \cdot c = 3 \cdot 8 = 24$ and add to $b = -10$. Then rewrite the middle term and factor by grouping.

- I need two numbers that multiply to _____ and add to _____.
- The two numbers are _____ and _____ because _____ times _____ equals 24 and _____ plus _____ equals negative 10.
- I rewrite the middle term: $3x^2 + ____x + ____x + 8$.
- Factoring by grouping:
  $3x^2 - 6x - 4x + 8 = 3x(x - ____) - 4(x - ____) = (3x - ____)(x - ____)$.

Answer: The two numbers are negative 6 and negative 4. They multiply to 24 and add to negative 10. Rewrite: $3x^2 - 6x - 4x + 8 = 3x(x - 2) - 4(x - 2) = (3x - 4)(x - 2)$. Check: $(3x - 4)(x - 2) = 3x^2 - 6x - 4x + 8 = 3x^2 - 10x + 8$.

Common error: Students may forget to factor out the common binomial in the last step or may choose factor pairs that multiply to 24 but do not add to negative 10. Stress that the middle-term rewrite must use both factor pairs and that the second binomial factor appears in both groups.

Question 3. Factor $2x^2 + 6x = 0$ by removing the greatest common factor first, then apply the zero-product property to find all solutions.

- The greatest common factor of $2x^2$ and $6x$ is _____.
- I factor out the GCF: $2x^2 + 6x = ____(x + ____)$.
- Using the zero-product property, either _____ or $(x + ____) = 0$.
- The two solutions are $x = ____$ and $x = ____$.

Answer: The greatest common factor is $2x$. Factor: $2x^2 + 6x = 2x(x + 3) = 0$. By the zero-product property, $2x = 0$ or $(x + 3) = 0$, so $x = 0$ or $x = -3$.

Common error: Students may forget to include $x = 0$ as a solution or may not recognize that the GCF is $2x$ (including the variable x). This problem addresses the error-analysis theme: dividing both sides by x would lose the $x = 0$ solution, but factoring preserves it.

Question 4. Solve $x^2 - 5x + 6 = 0$ by factoring. Show the factorization and use the zero-product property to justify both solutions.

- I need two numbers that multiply to _____ and add to _____.
- The two numbers are _____ and _____ because _____ times _____ equals 6 and _____ plus _____ equals negative 5.
- The factorization is $(x - ____)(x - ____) = 0$.
- By the zero-product property, $x - ____ = 0$ or $x - ____ = 0$, so $x = ____$ or $x = ____$.

Answer: The two numbers are negative 2 and negative 3. They multiply to 6 and add to negative 5. The factorization is $x^2 - 5x + 6 = (x - 2)(x - 3) = 0$. By the zero-product property, $(x - 2) = 0$ or $(x - 3) = 0$, so $x = 2$ or $x = 3$. Check: if $x = 2$, then $(2)^2 - 5(2) + 6 = 4 - 10 + 6 = 0$ ✓; if $x = 3$, then $(3)^2 - 5(3) + 6 = 9 - 15 + 6 = 0$ ✓.

This is the canonical exit-ticket problem. Common error: Students may choose the wrong sign or mix up which number goes with which factor. Emphasize checking both solutions by substitution into the original equation to verify correctness.

Packet B

Question 1. Factor the quadratic expression $x^2 + 8x + 15$ and then solve $x^2 + 8x + 15 = 0$ using the zero-product property. Justify each solution.

Answer: Factor: $x^2 + 8x + 15 = (x + 3)(x + 5)$. We look for two numbers that multiply to 15 and add to 8; those numbers are 3 and 5. Apply the zero-product property: If $(x + 3)(x + 5) = 0$, then $x + 3 = 0$ or $x + 5 = 0$. Solving each: $x = -3$ or $x = -5$. We justify $x = -3$ because when $x = -3$, the factor $(x + 3) = (-3 + 3) = 0$, making the product zero. We justify $x = -5$ because when $x = -5$, the factor $(x + 5) = (-5 + 5) = 0$, making the product zero.

Students often forget to include both solutions or to explicitly invoke the zero-product property. A common error is to state the factorization but not set each factor to zero. Ensure students write both equations $x + 3 = 0$ *and* $x + 5 = 0$ *and solve each separately.*

Question 2. Factor the quadratic expression $3x^2 - 10x + 8$ and then solve $3x^2 - 10x + 8 = 0$ using the zero-product property. Describe the strategy you used to find the factors.

Answer: Factor: $3x^2 - 10x + 8 = (3x - 4)(x - 2)$. Strategy: We find factor pairs of $a \cdot c = 3 \cdot 8 = 24$ that add to the middle coefficient -10. The pair -4 and -6 works because $(-4) \cdot (-6) = 24$ and $(-4) + (-6) = -10$. We split the middle term: $3x^2 - 4x - 6x + 8 = x(3x - 4) - 2(3x - 4) = (3x - 4)(x - 2)$. Apply the zero-product property: If $(3x - 4)(x - 2) = 0$, then $3x - 4 = 0$ or $x - 2 = 0$. Solving: $x = \frac{4}{3}$ or $x = 2$. The zero-product property justifies these solutions because when $x = \frac{4}{3}$, the factor $(3x - 4) = 0$; when $x = 2$, the factor $(x - 2) = 0$.

A frequent error is to attempt factoring by grouping but not correctly identify the factor pair of $a \cdot c$ that adds to b. Another common mistake is to forget to distribute correctly when splitting the middle term, or to lose the $\frac{4}{3}$ solution by incorrectly simplifying $3x - 4 = 0$. Remind students that $x = \frac{4}{3}$ is a valid solution and should be checked by substitution.

Question 3. A student solves the equation $x^2 = 6x$ by dividing both sides by x and obtains $x = 6$ as the only solution. Identify the error and solve the equation correctly using factoring and the zero-product property. Explain why the student's method caused the loss of a solution.

Answer: Error: Dividing by x is invalid when $x = 0$ because division by zero is undefined. This causes the loss of the solution $x = 0$. Correct solution: Rearrange to $x^2 - 6x = 0$. Factor out the greatest common factor: $x(x - 6) = 0$. Apply the zero-product property: $x = 0$ or $x - 6 = 0$, giving $x = 0$ or $x = 6$. Why factoring preserves both solutions: Factoring does not eliminate any values; it rewrites the equation so that the zero-product property can be applied to each factor separately. When we divide by x, we assume $x \neq 0$, which silently excludes the solution $x = 0$. Factoring reveals that x is a factor, forcing us to consider the case where $x = 0$ directly.

This is a critical conceptual error. Students often do not recognize that dividing by a variable eliminates solutions. Some may think that $x = 0$ is not a solution even after factoring. Check understanding by asking: 'Is $x = 0$ a solution to the original equation $x^2 = 6x$?' (Yes, because $0^2 = 0 = 6 \cdot 0$.) Emphasize that whenever a variable appears as a factor, it can be a solution and must never be divided away.

Question 4. Solve $x^2 - 5x + 6 = 0$ by factoring. Show the factorization and use the zero-product property to justify both solutions.

Answer: Factor: $x^2 - 5x + 6 = (x - 2)(x - 3)$. We find two numbers that multiply to 6 and add to -5; those numbers are -2 and -3. Apply the zero-product property: If $(x - 2)(x - 3) = 0$, then $x - 2 = 0$ or $x - 3 = 0$. Solving each: $x = 2$ or $x = 3$. Justification: By the zero-product property, if a product equals zero, at least one factor must equal zero. When $x = 2$, the factor $(x - 2) = 0$, so the product is zero. When $x = 3$, the factor $(x - 3) = 0$, so the product is zero. Both solutions satisfy the original equation.

This is the exit ticket question. Students must show the factorization clearly and explicitly name the zero-product property as justification for each solution. A common error is to find the factors correctly but fail to justify why setting each factor to zero yields valid solutions. Require students to state the zero-product property by name and apply it to both factors.

Packet C

Question 1. Analyze the following two strategies for solving $2x^2 + 7x + 3 = 0$. Strategy A: Factor by grouping to rewrite as $(2x + 1)(x + 3) = 0$, then apply the zero-product property. Strategy B: Divide both sides by x to eliminate the squared term, then solve. Evaluate which strategy preserves both solutions and explain the mathematical reasoning behind the success or failure of each approach.

Answer: Strategy A is the correct approach. The complete solution follows: $(2x + 1)(x + 3) = 0$ by factoring. Using the zero-product property, $2x + 1 = 0$ or $x + 3 = 0$, which gives $x = -\frac{1}{2}$ or $x = -3$. Strategy B fails because dividing both sides by x assumes $x \neq 0$, which eliminates any solution where $x = 0$. Although $x = 0$ is not a solution to this particular equation, dividing by a variable is mathematically invalid when solving quadratic equations because it risks losing solutions. The zero-product property is the only method that guarantees both solutions are found, since it requires the equation to be factored first and then each factor to be set equal to zero independently.

An exemplary response identifies that Strategy A uses the zero-product property correctly and recognizes that Strategy B dividing by x is invalid because it assumes $x \neq 0$ without justification. A strong student will note that even though $x = 0$ is not a solution here, the method is fundamentally flawed for quadratic equations in general. Students should explicitly state that factoring preserves all solutions because it does not restrict the domain.

Question 2. Create a quadratic equation of the form $ax^2 + bx + c = 0$ where $a > 1$ and the expression requires a greatest common factor to be factored first. Write the equation, factor it completely, apply the zero-product property to find both solutions, and then verify each solution by substitution. Explain why factoring out the greatest common factor first is essential to the success of your method.

Answer: Example equation: $3x^2 + 12x + 9 = 0$. Step 1: Factor out the greatest common factor: $3(x^2 + 4x + 3) = 0$. Step 2: Factor the monic quadratic: $3(x + 1)(x + 3) = 0$. Step 3: Apply the zero-product property. Since $3 \neq 0$, set $(x + 1) = 0$ and $(x + 3) = 0$, giving $x = -1$ and $x = -3$. Step 4: Verify by substitution. For $x = -1$: $3(-1)^2 + 12(-1) + 9 = 3 - 12 + 9 = 0$ ✓. For $x = -3$: $3(-3)^2 + 12(-3) + 9 = 27 - 36 + 9 = 0$ ✓. Factoring out the greatest common factor first simplifies the remaining quadratic and reduces computational error. If the greatest common factor is not removed, the factorization becomes more complex, and students may miss the structure of the monic quadratic hidden within.

An exemplary response demonstrates a valid quadratic with $a > 1$ and a non-trivial greatest common factor, shows complete factorization, correctly applies the zero-product property, and verifies both solutions by full substitution. The explanation should make clear that removing the greatest common factor reveals a simpler monic quadratic and that skipping this step would either complicate factorization or require dividing by the constant factor, which is inefficient.

Question 3. Evaluate the claim: 'The zero-product property is the only method that guarantees no solutions are lost when solving a quadratic equation.' Propose whether this claim is universally true or whether alternative methods (such as taking square roots or completing the square) could also preserve all solutions under certain conditions. Support your response with at least one specific example.

Answer: The claim is partially true but requires nuance. The zero-product property is the most reliable and direct method for factored quadratics because it sets each factor independently equal to zero, ensuring that all solutions corresponding to each factor are found. However, alternative methods can also preserve all solutions when applied correctly. For example, solving $x^2 - 5x + 6 = 0$ by completing the square: $x^2 - 5x + \frac{25}{4} = -6 + \frac{25}{4}$, which gives $(x - \frac{5}{2})^2 = \frac{1}{4}$. Taking the square root of both sides yields $x - \frac{5}{2} = \pm\frac{1}{2}$, so $x = 3$ or $x = 2$. Both solutions are preserved because the square root introduces the $\pm$ symbol. The risk of losing solutions arises when operations (such as dividing by a variable or squaring both sides without checking) restrict the domain or introduce extraneous solutions. The zero-product property is optimal for factored quadratics because it avoids these pitfalls by construction, but other methods are equally valid when executed with care.

An exemplary response acknowledges the truth of the claim for the factoring method while recognizing that other methods preserve solutions when properly applied. The student should demonstrate understanding that the risk of losing solutions is not inherent to all methods but rather to certain algebraic operations (such as dividing by a variable). A strong answer will provide a worked example showing that completing the square or another method yields both solutions, and will reflect on why factoring is preferred for quadratics that factor easily over the integers.

Question 4. Solve $x^2 - 5x + 6 = 0$ by factoring. Show the factorization and use the zero-product property to justify both solutions.

Answer: Step 1: Factor the quadratic. Identify two numbers that multiply to 6 and add to -5. These numbers are -2 and -3. Therefore, $x^2 - 5x + 6 = (x - 2)(x - 3)$. Step 2: Rewrite the equation in factored form: $(x - 2)(x - 3) = 0$. Step 3: Apply the zero-product property. If a product equals zero, then at least one factor must equal zero: $x - 2 = 0$ or $x - 3 = 0$. Step 4: Solve each equation. From $x - 2 = 0$, we get $x = 2$. From $x - 3 = 0$, we get $x = 3$. Step 5: Justify using the zero-product property. The zero-product property guarantees that the only values of x that make $(x - 2)(x - 3) = 0$ are $x = 2$ and $x = 3$, because these are the only values that make one of the factors equal to zero. Therefore, the solutions are $x = 2$ and $x = 3$.

A complete response must show the factorization, identify the correct factor pair, and apply the zero-product property to justify each solution. Verification by substitution is not required for the exit ticket but confirms correctness: $2^2 - 5(2) + 6 = 4 - 10 + 6 = 0$ ✓ *and* $3^2 - 5(3) + 6 = 9 - 15 + 6 = 0$ ✓.

Section 4: Teacher Toolkit

3-Point Mastery Rubric

3 — Complete Mastery: Meets all of these criteria accurately: Shows the factorization $x^2 - 5x + 6 = (x - 2)(x - 3)$ or an equivalent correct factorization.; Sets each factor equal to zero, $(x - 2) = 0$ and $(x - 3) = 0$, citing the zero-product property.; States both solutions, $x = 2$ and $x = 3$, and justifies each one using the zero-product property.

2 — Partial Understanding: Shows relevant understanding but partially meets or inaccurately addresses one or more of these criteria: Shows the factorization $x^2 - 5x + 6 = (x - 2)(x - 3)$ or an equivalent correct factorization.; Sets each factor equal to zero, $(x - 2) = 0$ and $(x - 3) = 0$, citing the zero-product property.; States both solutions, $x = 2$ and $x = 3$, and justifies each one using the zero-product property.

1 — Needs Significant Support: Shows limited evidence of meeting these criteria: Shows the factorization $x^2 - 5x + 6 = (x - 2)(x - 3)$ or an equivalent correct factorization.; Sets each factor equal to zero, $(x - 2) = 0$ and $(x - 3) = 0$, citing the zero-product property.; States both solutions, $x = 2$ and $x = 3$, and justifies each one using the zero-product property.

Common Misconceptions

Dividing by the variable loses a solution

Students think: Students believe that dividing both sides of $x^2 = 5x$ by x is always valid and yields the complete solution set $x = 5$.

Why: They treat division by x as a safe inverse operation and do not consider that x may equal 0, so the division silently discards the solution $x = 0$.

Fix: Have students rewrite the equation as $x^2 - 5x = 0$, factor to $x(x - 5) = 0$, and apply the zero-product property; then ask what value of x would make the original division invalid.

Confusing the factor pair with the solutions

Students think: Students report the two numbers that multiply to c and add to b as the solutions, giving $x = 2$ and $x = 3$ directly from the pair for $x^2 - 5x + 6$ without accounting for signs or setting factors to zero.

Why: They memorize a shortcut for monic quadratics and skip the step of writing factors as $(x - r)$ and solving each factor equation.

Fix: Require students to write the full factorization, then physically set each binomial equal to zero and solve, using the sentence stem 'The zero-product property tells me ___.' so the sign reversal is visible.

Skipping the greatest common factor

Students think: Students begin factoring $2x^2 + 6x$ by hunting for a factor pair of $2 \cdot 6$ instead of noticing that both terms share $2x$.

Why: They apply the $a \cdot c$ factoring routine mechanically without first checking for a common factor, which makes the numbers larger and the factoring error-prone.

Fix: Post a first-step checklist on the board: 'Is there a greatest common factor?' Have students factor out the greatest common factor before any other strategy and verify by expanding.

Implementation Notes

- Setup: Before students arrive, print Packet A, Packet B, and Packet C on different colors and place the writing workspace sheet with sentence stems at each desk. Post the two sentence stems 'I know the factors are ___ because ___.' and 'The zero-product property tells me ___.' on the board so students can reference them during pair work. Keep the optional early-finisher challenge card separate from the exit ticket stack so it is never confused with the summative check.
- Pacing: Follow the exact pacing outlined in Section 1's Lesson Flow, including the warm-up, Explore, error-analysis task, continued packet practice, synthesis, and the final Cool-down segment. Do not distribute the exit ticket until the Cool-down segment begins, and do not preview or collect it earlier. Use a visible timer for each segment so students can self-monitor their progress through the packet tasks.

- Grouping: Assign students to pairs heterogeneously so that each pair includes a student comfortable with factor pairs and a student who benefits from verbalizing the zero-product property. Assign Packet A to students needing the most scaffolding, Packet B to students ready for moderate support, and Packet C to students ready for less scaffolding, while keeping the same three core tasks in every packet. Circulate with the step-by-step answer key so you can confirm each pair's justification rather than only their final answers.
- Tiered packet flow: All three packets preserve the same learning goal and contain a monic quadratic, a quadratic with leading coefficient greater than 1, and a problem requiring a greatest common factor first. Packet A includes partially completed factor-pair tables and a factor, set-to-zero, check-by-substitution checklist; Packet B includes sentence stems with partial worked examples; Packet C includes open workspace with minimal prompts. During the error-analysis task, prompt every pair to name the lost solution $x = 0$ and to rewrite $x^2 - 5x = 0$ as $x(x - 5) = 0$ before applying the zero-product property.
- Monitoring and feedback: As pairs work, ask them to justify each factor pair aloud and to check each solution by substitution, recording both checks in the writing workspace. Collect exit tickets only at the end of the Cool-down segment and score them solely against the three success criteria for $x^2 - 5x + 6 = 0$. Use the optional early-finisher challenge only for students who complete their packet, and keep it separate from the exit ticket so it does not affect the summative score.

Extension Activities

- Early finisher challenge: Ask students to create and solve their own quadratic equation that requires a greatest common factor first, such as one of the form $ax^2 + bx = 0$ with $a \neq 0$, and to write a short explanation of why dividing both sides by the variable would lose a solution. Students should show the factorization, apply the zero-product property, and verify both solutions by substitution.
- Cross-curricular connection: Connect factoring to graphing by having students sketch the parabola $y = x^2 - 5x + 6$ and label its x-intercepts at $x = 2$ and $x = 3$, then write a sentence explaining how the factored form reveals the zeros of the function. This links the algebraic solutions to the graphical meaning of x-intercepts and supports the standard on revealing zeros of a function.
- Real-world application: Present a hypothetical scenario in which a rectangular garden has an area modeled by $x^2 + 7x + 12$ square units and side lengths that are binomials with integer coefficients; ask students to factor the expression to find possible side lengths and to explain why both factors must be positive in this context. Label the scenario as an invented classroom model rather than measured data, and have students justify each factor using the zero-product property when the area is set to zero.